Transverse mass as a means of measuring the W width at the Tevatron

David Summers

TH Division, CERN CH-1211 Geneva 23

Abstract

At the Tevatron the transverse mass is used to separate on mass shell from off mass shell W production; and the rate of off mass shell W production gives a measure of the W width. We look at alternative variables to see if the separation of on and off mass shell W's can be improved, and hence give a better measure of the W width. We find that the transverse mass is very close to the optimal variable for separating on from off mass shell W decay, and hence there is little to be gained by using other, more complicated, variables. This happens because if the transverse mass is above the W mass, the W is guaranteed to be produced off mass shell.

CERN-TH/96-205 August 1996 In the Standard Model of particle physics there are three massive vector bosons, the Z^0 and the W^\pm bosons. As these bosons are massive they have finite lifetimes and decay. The Z^0 boson, being electrically neutral, can be produced cleanly in e^+e^- annihilation. This means that its properties can be accurately measured. On the other hand the W bosons are electrically charged, and so cannot be produced in isolation in e^+e^- annihilations; instead currently real W bosons are produced in $p\bar{p}$ colliders, where the incoming partons can have net charge ± 1 . However $p\bar{p}$ colliders are not a clean environment in which to observe the W decay, where the decay via jets is typically hidden behind large QCD backgrounds. This leaves only the leptonic decay of the W to be observed; however, again because the W boson is charged, leptonic decays always involve an electrically neutral neutrino which goes undetected. Hence, although the properties of the Z boson and its decay are accurately measured, the properties of the W bosons and its decays are far less well known.

For the case of the W width no direct measurement can be made, instead there are currently two indirect methods of measuring the W width. In the first the ratio of dilepton Z events at the Tevatron is compared to single lepton + missing transverse energy W events [1]. We have

$$\frac{\sigma(pp \to W \to l\nu)}{\sigma(pp \to Z \to ll)} = \frac{\sigma(pp \to W)}{\sigma(pp \to Z)} \frac{\text{Br}(W \to l\nu)}{\text{Br}(Z \to ll)}$$

$$= \frac{\sigma(pp \to W)}{\sigma(pp \to Z)} \frac{\Gamma_Z}{\Gamma_W} \frac{\Gamma(W \to l\nu)}{\Gamma(Z \to ll)}.$$
(1)

Now $\frac{\sigma(pp\to W)}{\sigma(pp\to Z)}$ and $\frac{\Gamma(W\to l\nu)}{\Gamma(Z\to ll)}$ can be well predicted within perturbation theory; Γ_Z is accurately measured at LEP, and so this gives a measurement of Γ_W . Of course this assumes that $\frac{\sigma(pp\to W)}{\sigma(pp\to Z)}$ and $\frac{\Gamma(W\to l\nu)}{\Gamma(Z\to ll)}$ can be accurately predicted, which is in turn based upon assumptions such as that physics beyond the Standard Model does not modify these quantities.

In the second method the shape of the transverse mass M_T , spectrum of isolated lepton + missing energy W events, is measured [2, 3, 4, 5], where the transverse mass is defined by

$$M_T^2 = 2E_{T\nu}E_{Tl} - 2\mathbf{p}_{T\nu} \cdot \mathbf{p}_{Tl}.$$
 (2)

Now the transverse mass is always less than the actual mass, i.e.

$$p_W^2 \ge M_T^2; (3)$$

so, if the transverse mass is larger than M_W then the intermediate W must have been forced above its mass shell. Whereas on shell intermediate W bosons feel the effect of the W width in the Breit-Wigner propagator, for off shell ones the width term in the Breit-Wigner propagator is dominated by the standard term. This means that the rate of these off shell intermediate W bosons is proportional to the W width; hence the normalisation of the tail of the transverse mass distribution is sensitive to the W width Γ_W . This assumes that the leptonic decay of processes that take place via an off shell intermediate W can be related to those that proceed via an on shell intermediate W.

In this paper we study the second method, to see if it can be improved to give a more accurate determination of the W width. The crux of this method is the ability to separate on shell W production from processes where an off shell intermediate W is exchanged, and it is not immediately clear that M_T is the optimal variable to make such a separation. We have

$$p_W^2 = E_{T\nu} E_{Tl} \left(\exp(\Delta \eta) + \exp(-\Delta \eta) \right) - 2 \mathbf{p}_{T\nu} \cdot \mathbf{p}_{Tl}$$
$$= M_T^2 + E_{T\nu} E_{Tl} \left(\exp(\Delta \eta) + \exp(-\Delta \eta) - 2 \right), \tag{4}$$

where $\Delta \eta = |\eta_l - \eta_{\nu}|$. Now as the neutrino rapidity is unmeasured we cannot evaluate $\Delta \eta$ and hence not exactly reconstruct p_W^2 . When the unobserved neutrino has the same rapidity as the final state lepton M_T^2 equals p_W^2 ; as this is a vanishingly small part of phase space this clearly never exactly happens. Indeed it is not even clear, as lepton and transverse neutrino momenta vary from event to event, that M_T^2 has approximately the same behaviour as p_W^2 . As both M_T^2 and p_W^2 have the same dependence on $E_{T\nu}$ and E_{Tl} , and $\mathbf{p}_{T\nu}$ and \mathbf{p}_{Tl} are usually almost back to back, we shall concentrate on how M_T can be improved by measurements of the only remaining observed variable, the lepton rapidity η_l .

As M_T is relatively safe with respect to higher order corrections we shall use a leading order Monte Carlo simulation of W production at the Tevatron, that is a $p\bar{p}$ collider with $\sqrt{s} = 1.8$ TeV; for the W propagator we use a Breit-Wigner propagator with a running width term

$$Prop_W = \frac{1}{(p_W^2 - M_W^2)^2 + p_W^4 \Gamma_W^2 / M_W^2}$$
 (5)

which Dyson sums the imaginary part of the vacuum polarisation contribution via massless fermions to the W propagator. We replace the coupling in

the decay of the W boson into leptons by the W width,

$$g_W^2 = \frac{\Gamma_W^{\text{leptonic}}}{6\pi M_W} = \frac{\Gamma_W \text{Br}^{\text{TH}}(W \to l\nu)}{6\pi M_W}$$
 (6)

where $\mathrm{Br^{TH}}(W \to l\nu) = \Gamma_W^{\mathrm{TH-leptonic}}/\Gamma_W^{\mathrm{TH-total}} = 1/(9 + 6\alpha_S(M_W^2)/\pi) =$ 0.10810. Replacing the decay coupling constant ensures that in the narrow width approximation the overall cross-section to produce a W is independent of the W width Γ_W , as we physically expect (due to the physical independence of production and decay of a W). Notice that the use of BrTH rather than the unmeasured experimental value changes only the overall number of the W events; it does not change the shape of W distributions.

For the parton distributions we use MRS D0' [6] evaluated at a scale of $\sqrt{p_W^2}$, which at the Bjorken x and Q^2 values probed in W production at the Tevatron should give accurate results. For the remaining physical parameters we use the tree level Standard Model values with,

$$M_W = 80.22 \text{ GeV}$$
 (7)

$$\alpha = 1/128 \tag{8}$$

$$\alpha = 1/128 \tag{8}$$

$$\sin^2 \theta_w = 0.23. \tag{9}$$

Experimentally, measurements of W bosons at the Tevatron have large errors due to the unobserved neutrino in the leptonic decay; we model these errors by giving the measured missing transverse energy the normal distribution

$$\mathcal{P}(E) = \frac{\exp(-(E - E_{T\nu})^2 / (2 \text{ GeV}) E_{T\nu})}{\sqrt{2\pi E_{T\nu} / (\text{GeV})}},$$
(10)

relative to the transverse neutrino energy. Although this form of smearing is vastly simpler than the actual experimentally measured smearing of observables, it takes the dominant smearing into account.

We assume that the distribution of the smearing is known exactly; in practice we expect that this will be measured accurately in other processes such as Z decay, to the extent that it will only have a minimal effect on the W measurements.

As W^+ can be experimentally distinguished from W^- from the charge of the lepton with which it decays, and the W^- distributions are identical to the W^+ distributions under the transformation $\cos \theta \leftrightarrow -\cos \theta$, we plot W^- events reversing the sign of $\cos \theta$. This means that we gain extra information from the forward/backward asymmetry of the charged leptons, which would be symmetrised away if we did not distinguish the charges of the leptons.

We shall apply the same cuts on transverse energy as CDF use, i.e.

$$E_{Tl} > 30 \text{ GeV}$$
 $E_T > 30 \text{ GeV}$. (11)

We do not apply the CDF cut on the lepton rapidity ($|\eta_l| < 1.05$) as we retain this as a variable in all calculations [3].

We use two methods of measuring how capable different variables are of measuring the W width. All variables that we consider are sensitive to the W width in some regions where off mass shell W production dominates, and insensitive in other regions where on mass shell W production dominates. We estimate the region where off shell W production dominates by the region where the variable shows half or more of the full dependence on the W width; we then use the cross-section in this region as a measure of the ability of this variable to measure the W width. Now the cross-section in this region can be measured experimentally with an accuracy equal to the square root of the number of events in that region. We therefore expect to be able to measure the W width with an accuracy:

$$\frac{\Delta\Gamma_W}{\Gamma} \simeq \frac{1}{\sqrt{\sigma_{\text{off-shell}} \int \mathcal{L}}}.$$
 (12)

In the second method we generate 22739 unweighted W events that pass the cuts (11) with,

$$\Gamma_W = \Gamma_W^{\text{TH}} = \frac{\alpha_{\text{em}} M_W}{12 \sin^2 \theta_W} (9 + 6\alpha_S(M_W^2)) = 2.10 \text{ GeV};$$
 (13)

this corresponds to about $\int \mathcal{L} = 20 \text{ pb}^{-1}$. We then perform a binned log-likelihood fit for various values of the W width. The maximum of the log-likelihood gives the central estimate for the W width, and the region in which the log-likelihood drops by 0.5 gives the 1 standard deviation error (assuming that the errors are normal, or equivalently that the log-likelihood is parabolic). We always plot the log-likelihood relative to the maximum log-likelihood, as the absolute scale contains no physical information, depending

on factors such as the bin width. Just as CDF, we allow the normalisation of the differential cross-section to float, and only extract the log-likelihood from the shape of the differential cross-section. However, unlike CDF, we calculate the log-likelihood from the full differential cross-section, rather than just in the region where this is not sensitive to the experimental mismeasurement errors; as for our model, we know the effect of the experimental smearing exactly. This also saves considerable effort evaluating the region where we are insensitive to the experimental smearing for each variable that we consider.

We first compare the standard cross-section differential with respect to the transverse mass M_T with the best possible differential cross-section if the missing neutrino rapidity η_{ν} were known, that is the cross-section differential with respect to $\sqrt{p_W^2}$, where p_W^2 is defined by

$$p_W^2 = E_T E_{Tl} \left(\exp(\Delta \eta) + \exp(-\Delta \eta) \right) - 2 \mathbf{p}_T \cdot \mathbf{p}_{Tl}$$
 (14)

in Eq. (4) with $E_{T\nu}$ replaced by E_T .

In Figs. 1 and 2 we plot the event rate for the unweighted W events vs. the transverse mass M_T and the estimated momentum flowing through the W, $\sqrt{p_W^2}$ respectively. We also show the theoretical differential cross-section, normalised to the same number of events as the unweighted events, for the 3 values of the W width,

$$\Gamma_W = 1.5$$
 , 2.0 , 2.5 GeV. (15)

We show the dividing line, where the distribution shows half the full dependence on the W width. In the inset graph we show the log-likelihood vs. the W width. This gives 1σ errors in the measurement of the W width:

$$\Delta\Gamma_W^{M_T} = 0.103 \text{ GeV} \tag{16}$$

$$\Delta\Gamma_W^{p_W^2} = 0.067 \text{ GeV}. \tag{17}$$

The cross-sections in the off-shell regions are given by

$$\sigma_{\text{off-shell}}^{M_T} = 13.5 \text{ pb}$$
 (18)

$$\sigma_{\text{off-shell}}^{p_W^2} = 24.2 \text{ pb.} \tag{19}$$

For the transverse mass case the off-shell cross-section and 1σ errors agree well with Eq. (12); however, for the W invariant mass case the off-shell cross-section gives an error 50% larger than the log-likelihood error. This is because

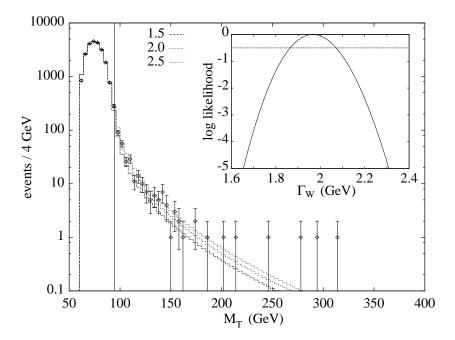


Figure 1: $d\sigma/dM_T$: for a sample of 22739 simulated events with $\Gamma_W = 2.10 \text{ GeV}$; the 3 theoretical curves are for $\Gamma_W = 1.5, 2.0, 2.5 \text{ GeV}$. Inset is the relative log-likelihood for different values of the W width.

the log-likelihood also gains some sensitivity to the W width from the region $p_W^2 < M_W^2$, i.e. from W's that are produced below mass shell; whereas the off-shell cross-section comes just from the region where the W is above its mass shell. This also explains why the W width extracted from the log-likelihood fit of the $\sqrt{p_W^2}$ spectrum is considerably lower than extracted from the M_T distribution. There are relatively few events with low p_W^2 , as $\sqrt{p_W^2}$ is sensitive to the W width in this region this drags the extracted width down. Whereas although there are also relatively few events at low M_T , because the low M_T region is not sensitive to the W width, this does not pull the extracted W width down. Nevertheless it is clear that the $\sqrt{p_W^2}$ spectrum gives at least 30% improvement in the W width measurement. In this paper we see if this improvement can be accessed.

Returning to Eq. (4) the first question to ask is how the average rapidity difference between the lepton and the neutrino varies as a function of the lepton rapidity. Naively we expect W's to be produced fairly centrally, so

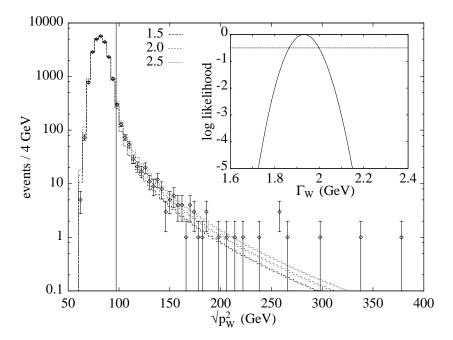


Figure 2: $d\sigma/d\sqrt{p_W^2}$: for a sample of 22739 simulated events with $\Gamma_W = 2.10$ GeV; the 3 theoretical curves are for $\Gamma_W = 1.5, 2.0, 2.5$ GeV. Inset is the relative log-likelihood for different values of the W width.

if the lepton is produced at large rapidities we expect the neutrino to be in the opposite hemisphere and $|\eta_l - \eta_{\nu}|$ to be large. This can be seen in the actual distribution, which is plotted in Fig. 3. Notice that with the cuts (11) an on-shell W can produce a lepton with maximum rapidity 3.91, with $|\eta_l - \eta_{\nu}| = 1.09$; as p_W^2 grows larger than M_W^2 , the maximum lepton rapidity grows slowly to 4.09, while the associated $|\eta_l - \eta_{\nu}|$ grows rapidly, as can be seen in Fig. 3.

For larger values of $|\eta_l|$, $|\eta_l - \eta_{\nu}|$ grows rapidly, which tells us that, for large values of $|\eta_l|$, the transverse mass significantly underestimates the momentum flowing through the W, especially in comparison to small values of $|\eta_l|$. This suggests using

$$p_{W,\text{est}}^2 = M_T^2 + E_T E_{Tl} \left(\exp(\overline{\Delta \eta}(\eta_l)) + \exp(-\overline{\Delta \eta}(\eta_l)) - 2 \right), \qquad (20)$$

where $\overline{\Delta\eta}(\eta_l)$ is the average value of $|\eta_l - \eta_{\nu}|$ shown in Fig. 3.

We show the analogous graph to Figs. 1 and 2 for the variable $\sqrt{p_{W,\mathrm{est}}^2}$ in

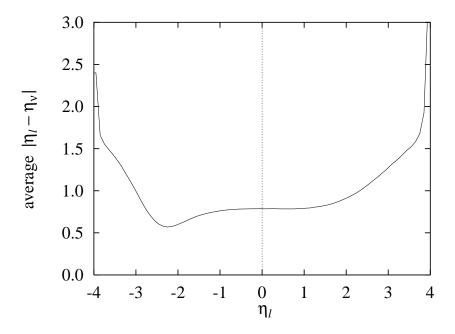


Figure 3: The average value of $|\eta_l - \eta_{\nu}|$ vs. η_l for W events.

Fig. 4. The log-likelihood gives a measurement of the statistical error in the W width of

$$\Delta \Gamma_{W,\text{est}}^{p_{W,\text{est}}^2} = 0.103 \text{ GeV}. \tag{21}$$

The cross-section in the off-shell regions is given by

$$\sigma_{\rm off-shell}^{p_{W,\rm est}^2} = 13.3 \text{ pb.} \tag{22}$$

Clearly $\sqrt{p_{W,\mathrm{est}}^2}$ is no better a variable than M_T in separating on- from off-shell W production, and it has given almost identical errors. This is because although $\sqrt{p_{W,\mathrm{est}}^2}$ is a far better estimator of the momentum flowing through the W (it clearly peaks far closer to the W mass than M_T), the region where we are sensitive to the W width clearly moves up to a far higher value. For M_T values greater than 95 GeV, the differential cross-section shows a more than 50% dependence on the W width, whereas for $\sqrt{p_{W,\mathrm{est}}^2}$ values greater than 102 GeV are needed until we have a more than 50% dependence on the W width. If there is no experimental smearing, then $M_T^2 \leq p_W^2$ guarantees that the exchanged W is off mass shell if the transverse mass is greater than

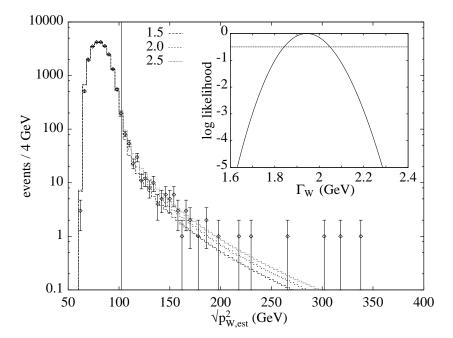


Figure 4: $d\sigma/d\sqrt{p_{W,\rm est}^2}$: for a sample of 22739 simulated events with $\Gamma_W = 2.10$ GeV; the 3 theoretical curves are for $\Gamma_W = 1.5, 2.0, 2.5$ GeV. Inset is the relative log-likelihood for different values of the W width.

 M_W ; whereas $p_{W,\mathrm{est}}^2 \simeq p_W^2$ and that on-mass shell W production dominates off-mass shell production, means that for $p_{W,\mathrm{est}}^2$ values just above the W mass squared are most likely to be from on mass shell W production where $p_{W,\mathrm{est}}^2$ overestimates the momentum flowing through the W. To counteract this effect, $p_{W,\mathrm{est}}^2$ has to be larger than M_T^2 before the cross-section becomes sensitive to the W width.

Clearly we should not be interested in a more accurate determination of p_W^2 , as the unknown neutrino rapidity means that we can only reconstruct the p_W^2 of an ensemble of W decays; in order to measure the W width we need to evaluate p_W^2 on an event by event basis. With this in mind, we ask a different question than what is the average value of $|\eta_l - \eta_{\nu}|$ for different η_l values; what we are more interested in is how the sensitivity to the W width varies with the lepton rapidity. To be sensitive to the W width we need the W to be off mass shell, that is $\sqrt{p_W^2} - M_W \gtrsim \Gamma_W$. In Fig. 5 we show the fraction of events that have $\sqrt{p_W^2} > M_W + \Gamma_W$ as a function both of η_l and

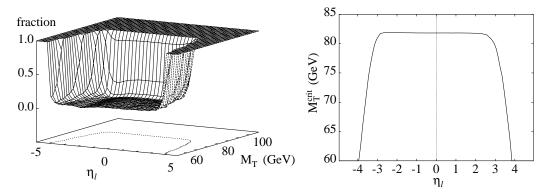


Figure 5: The fraction of W events that have $\sqrt{p_W^2} > M_W + \Gamma_W$ vs. M_T and η_l .

 M_T ; we also show the M_T required for each η_l , such that 50% of the events have $\sqrt{p_W^2} > M_W + \Gamma_W$.

What we require is a variable such that for all values of the lepton rapidity the variable becomes sensitive to the W width at the same value. From Fig. 5 it is clear that this does not happen for the transverse mass, although in the central region, with $|\eta_l| < 2.5$, the transverse mass becomes sensitive to the W width at approximately the constant value of just above the W mass. A simple modification of M_T that has the property of becoming sensitive to the W width for the same value, independent of the lepton rapidity, is

$$M_T^{\text{mod}} = M_T \frac{M_W}{M_T^{\text{crit}}(\eta_l)} \tag{23}$$

where M_T^{crit} is obtained from Fig. 5. We scale by M_W in the numerator, so $M_T^{\text{mod}} \approx M_T$ for the central region. We show the analogous graph to Figs. 1 and 2 for the variable M_T^{mod} in Fig. 6. The log-likelihood gives a measurement of the statistical error in the W width of

$$\Delta\Gamma_W^{M_T^{\rm mod}} = 0.103 \text{ GeV}. \tag{24}$$

The cross-section in the off-shell regions is given by

$$\sigma_{\text{off-shell}}^{M_T^{\text{mod}}} = 13.5 \text{ pb.} \tag{25}$$

It can be seen that M_T^{mod} does not lead to a significant improvement in the measurement of the W width. This happens because M_T^{mod} is essentially

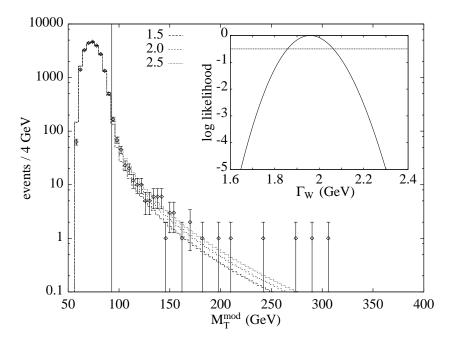


Figure 6: $d\sigma/dM_T^{\rm mod}$: for a sample of 22739 simulated events with $\Gamma_W = 2.10$ GeV; the 3 theoretical curves are for $\Gamma_W = 1.5, 2.0, 2.5$ GeV. Inset is the relative log-likelihood for different values of the W width.

identical to M_T for $|\eta_l| < 2.5$, and the vast majority of W's are produced with $|\eta_l| < 2.5$. This also means that if there is an experimental cut on the lepton rapidity, such as $|\eta_l| < 1.05$ that CDF apply, then M_T is effectively the optimal variable for separating on- from off-shell W decays; certainly there is little to be gained from measurements of the lepton rapidity.

Conclusions

In this paper we look at improvements that can be made to the transverse mass variable at the Tevatron to enhance measurements of the W width. In particular we look at enhancements that come from using the rapidity of the measured charged lepton that comes from the W decay. We construct 2 new variables, $p_{W,\text{est}}^2$, which estimates the momentum flowing through the W, and M_T^{mod} , which for all charged lepton rapidity values becomes sensitive to the W width at the same value. $p_{W,\text{est}}^2$ does not enhance measurements of the W

width, as the region where this variable becomes sensitive to the W width is moved to higher values, where the cross-section is lower.

On the other hand, $M_T^{\rm mod}$ is constructed to be the optimal modification of the transverse mass from measuring the charged lepton rapidity. However, for values of $|\eta_l| < 2.5$, $M_T^{\rm mod}$ and the transverse mass are effectively equivalent. Typically there are very few events with $|\eta_l| > 2.5$; this means that $M_T^{\rm mod}$ leads to no significant improvement in the measurement of the W width. This is especially true if there is an experimental cut on the charged lepton rapidity.

Acknowledgements

I would like to thank Mike Seymour for helpful comments on the manuscript.

References

- [1] CDF Collaboration, F. Abe et al., Phys. Rev. Lett. 73 (1994) 220.
- [2] J. Smith, W.L. van Neerven and J.A.M. Vermaseren Phys. Rev. Lett. **50** (1983) 1738.
- [3] CDF Collaboration, F. Abe et al., Phys. Rev. Lett. 74 (1995) 342.
- [4] C. Albajar et al., Z. Phys. C44 (1989) 15.
- [5] R. Ansari et al., Phys. Lett. B **186** (1987) 440.
- [6] A. D. Martin, R. G. Roberts and W. J. Stirling, Phys. Lett. B306 (1993) 145.